GCE AS/A LEVEL

wjec cbac

WJEC GCE AS/A Level in

MATHEMATICS

APPROVED BY QUALIFICATIONS WALES

SPECIFICATION

Teaching from 2017

For award from 2018 (AS)
For award from 2018 (A level)

Version 2 March 2019

This Qualifications Wales regulated qualification is not available to centres in England.

SUMMARY OF AMENDMENTS

Version	Description	Page number
2	'Making entries' section has been amended to clarify resit rules.	38

WJEC GCE AS and A LEVEL in MATHEMATICS

For teaching from 2017
For AS award from 2018
For A level award from 2018

This specification meets the Approval Criteria for GCE AS and A Level Mathematics and the GCE AS and A Level Qualification Principles which set out the requirements for all new or revised GCE specifications developed to be taught in Wales from September 2017.

GCE AS and A LEVEL MATHEMATICS (Wales)
SUMMARY OF ASSESSMENT

This specification is divided into a total of 4 units, 2 AS units and 2 A2 units. Weightings noted below are expressed in terms of the full A level qualification.

All units are compulsory.

AS (2 units)

AS Unit 1: Pure Mathematics A Written examination: 2 hours 30 minutes 25% of qualification	120 marks
The paper will comprise a number of short and longer, both structured and unstructured questions, which may be set on any part of the subject content of the unit. A number of questions will assess learners' understanding of more than one topic from the subject content. A calculator will be allowed in this examination.	
AS Unit 2: Applied Mathematics A Written examination: 1 hour 45 minutes 15% of qualification	75 marks
The paper will comprise two sections: **Section A: Statistics (40 marks)** **Section B: Mechanics (35 marks)** The total assessment time of 1 hour 45 minutes can be split between Section A and Section B as learners deem appropriate. The paper will comprise a number of short and longer, both structured and unstructured questions, which may be set on any part of the subject content of the unit. A number of questions will assess learners' understanding of more than one topic from the subject content. A calculator will be allowed in this examination.	

A Level (the above plus a further 2 units)

A2 Unit 3: Pure Mathematics B Written examination: 2 hours 30 minutes 35% of qualification 120 marks
The paper will comprise a number of short and longer, both structured and unstructured questions, which may be set on any part of the subject content of the unit. A number of questions will assess learners' understanding of more than one topic from the subject content. A calculator will be allowed in this examination.
A2 Unit 4: Applied Mathematics B Written examination: 1 hour 45 minutes 25% of qualification 80 marks
The paper will comprise two sections: **Section A: Statistics (40 marks)** **Section B: Differential Equations and Mechanics (40 marks)** The total assessment time of 1 hour 45 minutes can be split between Section A and Section B as learners deem appropriate. The paper will comprise a number of short and longer, both structured and unstructured questions, which may be set on any part of the subject content of the unit. A number of questions will assess learners' understanding of more than one topic from the subject content. A calculator will be allowed in this examination.

This is a unitised specification, which allows for an element of staged assessment. Assessment opportunities will be available in the summer assessment period each year, until the end of the life of the specification.

Unit 1 and Unit 2 will be available in 2018 (and each year thereafter) and the AS qualification will be awarded for the first time in summer 2018.

Unit 3 and Unit 4 will be available in 2018 (and each year thereafter) and the A level qualification will be awarded for the first time in summer 2018.

Qualification Number listed on The Register:
GCE AS: 603/1983/5
GCE A level: 603/1977/X

Qualifications Wales Approval Number listed on QiW:
GCE AS: C00/1173/7
GCE A level: C00/1153/6

GCE AS AND A LEVEL MATHEMATICS

1 INTRODUCTION

1.1 Aims and objectives

This WJEC GCE AS and A level in Mathematics provides a broad, coherent, satisfying and worthwhile course of study. It encourages learners to develop confidence in, and a positive attitude towards, mathematics and to recognise its importance in their own lives and to society. The specification has been designed to respond to the proposals set out in the report of the ALCAB panel on mathematics and further mathematics.

The WJEC GCE AS and A level in Mathematics encourages learners to:

- develop their understanding of mathematics and mathematical processes in a way that promotes confidence and fosters enjoyment;
- develop abilities to reason logically and recognise incorrect reasoning, to generalise and to construct mathematical proofs;
- extend their range of mathematical skills and techniques and use them in more difficult, unstructured problems;
- develop an understanding of coherence and progression in mathematics and of how different areas of mathematics can be connected;
- recognise how a situation may be represented mathematically and understand the relationship between ‘real world’ problems and standard and other mathematical models and how these can be refined and improved;
- use mathematics as an effective means of communication;
- read and comprehend mathematical arguments and articles concerning applications of mathematics;
- acquire the skills needed to use technology such as calculators and computers effectively, recognise when such use may be inappropriate and be aware of limitations;
- develop an awareness of the relevance of mathematics to other fields of study, to the world of work and to society in general;
- take increasing responsibility for their own learning and the evaluation of their own mathematical development.

1.2 Prior learning and progression

Any requirements set for entry to a course following this specification are at the discretion of centres. It is reasonable to assume that many learners will have achieved qualifications equivalent to Level 2 at KS4. Skills in Numeracy/Mathematics, Literacy/English and Information and Communication Technology will provide a good basis for progression to this Level 3 qualification.

This specification builds on the knowledge, understanding and skills established at GCSE.

This specification provides a suitable foundation for the study of mathematics or a related area through a range of higher education courses, progression to the next level of vocational qualifications or employment. In addition, the specification provides a coherent, satisfying and worthwhile course of study for learners who do not progress to further study in this subject.

This specification is not age specific and, as such, provides opportunities for learners to extend their life-long learning.

1.3 Equality and fair access

This specification may be followed by any learner, irrespective of gender, ethnic, religious or cultural background. It has been designed to avoid, where possible, features that could, without justification, make it more difficult for a learner to achieve because they have a particular protected characteristic.

The protected characteristics under the Equality Act 2010 are age, disability, gender reassignment, pregnancy and maternity, race, religion or belief, sex and sexual orientation.

The specification has been discussed with groups who represent the interests of a diverse range of learners, and the specification will be kept under review.

Reasonable adjustments are made for certain learners in order to enable them to access the assessments (e.g. candidates are allowed access to a Sign Language Interpreter, using British Sign Language). Information on reasonable adjustments is found in the following document from the Joint Council for Qualifications (JCQ): *Access Arrangements and Reasonable Adjustments: General and Vocational Qualifications.*

This document is available on the JCQ website (www.jcq.org.uk). As a consequence of provision for reasonable adjustments, very few learners will have a complete barrier to any part of the assessment.

1.4 Welsh Baccalaureate

In following this specification, learners should be given opportunities, where appropriate, to develop the skills that are being assessed through the Skills Challenge Certificate within the Welsh Baccalaureate:

- Literacy
- Numeracy
- Digital Literacy
- Critical Thinking and Problem Solving
- Planning and Organisation
- Creativity and Innovation
- Personal Effectiveness.

1.5 Welsh perspective

In following this specification, learners should be given opportunities, where appropriate, to consider a Welsh perspective if the opportunity arises naturally from the subject matter and if its inclusion would enrich learners' understanding of the world around them as citizens of Wales as well as the UK, Europe and the world.

2 SUBJECT CONTENT

Mathematics is, inherently, a sequential subject. There is a progression of material through all levels at which the subject is studied. The specification content therefore builds on the skills, knowledge and understanding set out in the whole GCSE subject content for Mathematics and Mathematics-Numeracy for first teaching from 2015.

Overarching themes

This GCE AS and A Level specification in Mathematics requires learners to demonstrate the following overarching knowledge and skills. These must be applied, along with associated mathematical thinking and understanding, across the whole of the detailed content set out below. The knowledge and skills required for AS Mathematics are shown in bold text. The text in standard type applies to A2 only.

Mathematical argument, language and proof

GCE AS and A Level Mathematics specifications must use the mathematical notation set out in Appendix A and must require learners to recall the mathematical formulae and identities set out in Appendix B.

Knowledge/Skill
Construct and present mathematical arguments through appropriate use of diagrams; sketching graphs; logical deduction; precise statements involving correct use of symbols and connecting language, including: constant, coefficient, expression, equation, function, identity, index, term, variable
Understand and use mathematical language and syntax as set out in the content
Understand and use language and symbols associated with set theory, as set out in the content. **Apply to solutions of inequalities and probability**
Understand and use the definition of a function; domain and range of functions
Comprehend and critique mathematical arguments, proofs and justifications of methods and formulae, including those relating to applications of mathematics

Mathematical problem solving

Knowledge/Skill
Recognise the underlying mathematical structure in a situation and simplify and abstract appropriately to enable problems to be solved
Construct extended arguments to solve problems presented in an unstructured form, including problems in context
Interpret and communicate solutions in the context of the original problem
Understand that many mathematical problems cannot be solved analytically, but numerical methods permit solution to a required level of accuracy
Evaluate, including by making reasoned estimates, the accuracy or limitations of solutions, including those obtained using numerical methods
Understand the concept of a mathematical problem solving cycle, including specifying the problem, collecting information, processing and representing information and interpreting results, which may identify the need to repeat the cycle
Understand, interpret and extract information from diagrams and construct mathematical diagrams to solve problems, including in mechanics

Mathematical modelling

Knowledge/Skill
Translate a situation in context into a mathematical model, making simplifying assumptions
Use a mathematical model with suitable inputs to engage with and explore situations (for a given model or a model constructed or selected by the learner)
Interpret the outputs of a mathematical model in the context of the original situation (for a given model or a model constructed or selected by the learner)
Understand that a mathematical model can be refined by considering its outputs and simplifying assumptions; evaluate whether the model is appropriate
Understand and use modelling assumptions

Use of data in statistics

This specification requires learners, during the course of their study, to:

- develop skills relevant to exploring and analysing large data sets (these data must be real and sufficiently rich to enable the concepts and skills of data presentation and interpretation in the specification to be explored);
- use technology such as spreadsheets or specialist statistical packages to explore data sets;
- interpret real data presented in summary or graphical form;
- use data to investigate questions arising in real contexts.

Learners should be able to demonstrate the ability to explore large data sets, and associated contexts, during their course of study to enable them to perform tasks, and understand ways in which technology can help explore the data. Learners should be able to demonstrate the ability to analyse a subset or features of the data using a calculator with standard statistical functions.

2.1 AS UNIT 1

Unit 1: Pure Mathematics A
Written examination: 2 hours 30 minutes
25% of A level qualification (62.5% of AS qualification)
120 marks

The subject content is set out on the following pages. There is no hierarchy implied by the order in which the content is presented, nor should the length of the various sections be taken to imply any view of their relative importance.

Topics	Guidance
2.1.1 Proof	
Understand and use the structure of mathematical proof, proceeding from given assumptions through a series of logical steps to a conclusion; use methods of proof, including (a) proof by deduction, (b) proof by exhaustion, (c) disproof by counter example.	Proof by deduction to include the proofs of the laws of logarithms.
2.1.2 Algebra and Functions	
Understand and use the laws of indices for all rational exponents. Use and manipulate surds, including rationalising the denominator.	To include rationalising fractions such as $\dfrac{2+3\sqrt{5}}{3-2\sqrt{5}}$ and $\dfrac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}}$.
Work with quadratic functions and their graphs. The discriminant of a quadratic function, including the conditions for real roots and repeated roots. Completing the square. Solution of quadratic equations in a function of the unknown.	The nature of the roots of a quadratic equation. To include finding the maximum or minimum value of a quadratic function. To include by factorisation, use of the formula and completing the square.

Topics	Guidance
Solve simultaneous equations in two variables by elimination and by substitution, including one linear and one quadratic equation.	To include finding the points of intersection or the point of contact of a line and a curve.
Solve linear and quadratic inequalities in a single variable and interpret such inequalities graphically, including inequalities with brackets and fractions. Express solutions through the correct use of 'and' and 'or', or through set notation. Represent linear and quadratic inequalities graphically.	To include the solution of inequalities such as $1 - 2x < 4x + 7$, $\frac{x}{2} \geq 2(1-3x)$ and $x^2 - 6x + 8 \geq 0$. To include, for example, $y > x+1$ (a strict inequality) and $y \geq ax^2 + bx + c$ (a non-strict inequality).
Manipulate polynomials algebraically, including expanding brackets and collecting like terms, factorisation and simple algebraic division; use of the Factor Theorem.	The use of the Factor Theorem will be restricted to cubic polynomials and the solution of cubic equations.
Understand and use graphs of functions; sketch curves defined by simple equations, including polynomials. $y = \frac{a}{x}$ and $y = \frac{a}{x^2}$, including their vertical and horizontal asymptotes. Interpret algebraic solutions of equations graphically. Use intersection points of graphs of curves to solve equations. Understand and use proportional relationships and their graphs.	The equations will be restricted to the form $y = f(x)$.
Understand the effect of simple transformations on the graph of $y = f(x)$ including sketching associated graphs: $y = af(x)$, $y = f(x) + a$, $y = f(x + a)$, $y = f(ax)$.	

Topics	Guidance
2.1.3 Coordinate geometry in the (x, y) plane	
Understand and use the equation of a straight line, including the forms $y = mx + c$, $y - y_1 = m(x - x_1)$ and $ax + by + c = 0$; gradient conditions for two straight lines to be parallel or perpendicular. Be able to use straight line models in a variety of contexts.	To include • finding the gradient, equation, length and midpoint of a line joining two given points; • the equations of lines which are parallel or perpendicular to a given line.
Understand and use the coordinate geometry of the circle using the equation of a circle in the form $(x - a)^2 + (y - b)^2 = r^2$; completing the square to find the centre and radius of a circle. Use of the following circle properties: (i) the angle in a semicircle is a right angle; (ii) the perpendicular from the centre to a chord bisects the chord; (iii) the radius of a circle at a given point on its circumference is perpendicular to the tangent to the circle at that point.	To also be familiar with the equation of a circle in the form $x^2 + y^2 + 2gx + 2fy + c = 0$. To include: • finding the equations of tangents, • the condition for two circles to touch internally or externally, • finding the points of intersection or the point of contact of a line and a circle,
2.1.4 Sequences and Series - The Binomial Theorem	
Understand and use the binomial expansion of $(a + bx)^n$ for positive integer n . The notations $n!$, $\binom{n}{r}$ and $n\mathrm{C}r$. Link to binomial probabilities.	To include use of Pascal's triangle.

Topics	Guidance
2.1.5 Trigonometry	
Understand and use the definitions of sine, cosine and tangent for all arguments.	Use of the exact values of the sine, cosine and tangent of 30°, 45° and 60°.
Understand and use the sine and cosine rules, and the area of a triangle in the form ½ $ab\sin C$.	To include the use of the sine rule in the ambiguous case.
Understand and use the sine, cosine and tangent functions. Understand and use their graphs, symmetries and periodicity.	
Understand and use $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$. Understand and use $\cos^2\theta + \sin^2\theta = 1$.	These identities may be used to solve trigonometric equations or prove trigonometric identities.
Solve simple trigonometric equations in a given interval, including quadratic equations in $\sin$, $\cos$ and $\tan$, and equations involving multiples of the unknown angle.	To include the solution of equations such as $3\sin\theta = 1$, $\tan\theta = \dfrac{\sqrt{3}}{2}$, $3\cos 2\theta = -1$ and $2\cos^2\theta + \sin\theta - 1 = 0$.
2.1.6 Exponentials and logarithms	
Know and use the function a^x and its graph, where a is positive. Know and use the function e^x and its graph.	
Know that the gradient of e^{kx} is equal to ke^{kx} and hence understand why the exponential model is suitable in many applications.	Realise that when the rate of change is proportional to the y value, an exponential model should be used.

Topics	Guidance
Know and use the definition of $\log_a x$ as the inverse of a^x, where a is positive and $x \geq 0$. Know and use the function $\ln x$ and its graph. Know and use $\ln x$ as the inverse function of e^x.	
Understand and use the laws of logarithms. $\log_a x + \log_a y = \log_a (xy)$ $\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$ $k \log_a x = \log_a (x^k)$ (including, for example $k = -1$, $k = -½$)	To include the proof of the laws of logarithms. Use of the laws of logarithms. e.g. Simplify $\log_2 36 - 2\log_2 15 + \log_2 100 + 1$. Change of base will not be required.
Solve equations in the form $a^x = b$.	The use of a calculator to solve equations such as (i) $3^x = 2$, (ii) $25^x - 4 \times 5^x + 3 = 0$. (iii) $4^{2x+1} = 5^x$
Use logarithmic graphs to estimate parameters in relationships of the form $y = ax^n$ and $y = kb^x$, given data for x and y.	Link to laws of logarithms. Understand that on a graph of $\log y$ against $\log x$, the gradient is n and the intercept is $\log a$, and that on a graph of $\log y$ against x, the gradient is $\log b$ and the intercept is $\log k$.
Understand and use exponential growth and decay; use in modelling (examples may include the use of e in continuous compound interest, radioactive decay, drug concentration decay, exponential growth as model for population growth.) Consideration of limitations and refinements of exponential models.	The formal differentiation and integration of formulae involving e^x and/or a^x will not be required.

Topics	Guidance
2.1.7 Differentiation	
Understand and use the derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a general point (x, y); the gradient of the tangent as a limit; interpretation as a rate of change; sketching the gradient function for a given curve; second order derivatives. Differentiation from first principles for small positive integer powers of x. Understand and use the second derivative as the rate of change of gradient.	The notation $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $f'(x)$ may be used. Up to and including power of 3. To include polynomials up to and including a maximum degree of 3.
Differentiate x^n for rational n, and related constant multiples, sums and differences.	To include polynomials.
Apply differentiation to find gradients, tangents and normals, maxima and minima, and stationary points. Identify where functions are increasing or decreasing.	To include finding the equations of tangents and normals. The use of maxima and minima in simple optimisation problems. To include simple curve sketching.
2.1.8 Integration	
Know and use the Fundamental Theorem of Calculus.	Integration as the reverse of differentiation.
Integrate x^n (excluding $n = -1$) and related sums, differences and constant multiples.	To include polynomials.
Evaluate definite integrals. Use a definite integral to find the area under a curve.	To include finding the area of a region between a straight line and a curve.

Topics	Guidance
2.1.9 Vectors	
Use vectors in two dimensions.	To include the use of the unit vectors, $\mathbf{i}$ and $\mathbf{j}$.
Add vectors diagrammatically and perform the algebraic operations of vector addition and multiplication by scalars, and understand their geometrical interpretations.	Condition for two vectors to be parallel.
Understand and use position vectors; calculate the distance between points represented by position vectors. Use vectors to solve problems in pure mathematics.	Use of $\mathbf{AB} = \mathbf{b} - \mathbf{a}$. To include the use of position vectors given in terms of unit vectors. To include the use and derivation of the position vector of a point dividing a line in a given ratio.

2.2 AS UNIT 2

Unit 2: Applied Mathematics A
Written examination: 1 hour 45 minutes
15% of A level qualification (37.5% of AS qualification)
75 marks

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in Unit 1.

The paper will comprise two sections:

Section A: Statistics (40 marks)

Section B: Mechanics (35 marks)

The total assessment time of 1 hour 45 minutes can be split between Section A and Section B as candidates deem appropriate.

The subject content is set out on the following pages. There is no hierarchy implied by the order in which the content is presented, nor should the length of the various sections be taken to imply any view of their relative importance.

Topics	Guidance
STATISTICS	
2.2.1 Statistical Sampling	
Understand and use the terms 'population' and 'sample'. Use samples to make informal inferences about the population. Understand and use sampling techniques, including simple random sampling, systematic sampling and opportunity sampling.	
Select or critique sampling techniques in the context of solving a statistical problem, including understanding that different samples can lead to different conclusions about the population.	

Topics	Guidance
2.2.2 Data presentation and interpretation	
Interpret diagrams for single-variable data, including understanding that area in a histogram represents frequency. Connect to probability distributions.	Learners should be familiar with box and whisker diagrams and cumulative frequency diagrams. Qualitative assessment of skewness is expected and the use of the terms symmetric, positive skew or negative skew
Interpret scatter diagrams and regression lines for bivariate data, including recognition of scatter diagrams which include distinct sections of the population. (Calculations of coefficients of regression lines are excluded.) Understand informal interpretation of correlation. Understand that correlation does not imply causation.	Equations of regression lines may be given in a question and learners asked to make predictions using it. Use of the terms positive, negative, zero, strong and weak is expected.
Interpret measures of central tendency and variation, extending to standard deviation. Be able to calculate standard deviation, including from summary statistics.	Measures of central tendency: mean, median, mode. Measures of central variation: variance, standard deviation, range, interquartile range.
Recognise and interpret possible outliers in data sets and statistical diagrams. Select or critique data presentation techniques in the context of a statistical problem. Be able to clean data, including dealing with missing data, errors and outliers.	Use of $Q_1 - 1.5 \times IQR$ and $Q_3 + 1.5 \times IQR$ to identify outliers.

Topics	Guidance
2.2.3 Probability	
Understand and use mutually exclusive and independent events when calculating probabilities. Link to discrete and continuous distributions.	To include the multiplication law for independent events: $\mathrm{P}(A \cap B) = \mathrm{P}(A)\,\mathrm{P}(B)$.
Use Venn diagrams to calculate probabilities.	Use of set notation and associated language is expected. To include the generalised addition law: $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$. Conditional probability will not be assessed in this unit.
2.2.4 Statistical distributions	
Understand and use simple, discrete probability distributions. Understand and use, • the binomial distribution, as a model • the Poisson distribution, as a model • the discrete uniform distribution, as a model (Calculation of mean and variance of discrete random variables is excluded.)	To include using distributions to model real world situations and to comment on their appropriateness.
Calculate probabilities using • the binomial distribution. • the Poisson distribution. • the discrete uniform distribution.	Use of the binomial formula and tables / calculator. Use of the Poisson formula and tables / calculator Use of the formula for the discrete uniform distribution.

Topics	Guidance
Select an appropriate probability distribution for a context, with appropriate reasoning, including recognising when the binomial, Poisson or discrete uniform model may not be appropriate.	
2.2.5 Statistical hypothesis testing	
Understand and apply the language of statistical hypothesis testing, developed through a binomial model: null hypothesis, alternative hypothesis, significance level, test statistic, 1-tail test, 2-tail test, critical value, critical region, acceptance region, *p*-value.	The *p*-value is the probability that the observed result or a more extreme one will occur under the null hypothesis H_0. For uniformity, interpretations of a *p*-value should be along the following lines: $p < 0{\cdot}01$; there is very strong evidence for rejecting H_0. $0{\cdot}01 \le p \le 0{\cdot}05$; there is strong evidence for rejecting H_0. $p > 0{\cdot}05$; there is insufficient evidence for rejecting H_0.
Conduct a statistical hypothesis test for the proportion in the binomial distribution and interpret the results in context. Understand that a sample is being used to make an inference about the population and appreciate that the significance level is the probability of incorrectly rejecting the null hypothesis.	
Interpret and calculate Type I and Type II errors, and know their practical meaning.	

Topics	Guidance
MECHANICS	
2.2.6 Quantities and units in mechanics	
Understand and use fundamental quantities and units in the S.I. system; length, time and mass.	
Understand and use derived quantities and units: velocity, acceleration, force, weight.	
2.2.7 Kinematics	
Understand and use the language of kinematics: position, displacement, distance travelled, velocity, speed, acceleration.	
Understand, use and interpret graphs in kinematics for motion in a straight line: displacement against time and interpretation of the gradient; velocity against time and interpretation of the gradient and the area under the graph	Learners may be expected to sketch displacement-time and velocity-time graphs.
Understand, use and derive the formulae for constant acceleration for motion in a straight line.	To include vertical motion under gravity. Gravitational acceleration, g. The inverse square law for gravitation is not required and g may be assumed to be constant, but learners should be aware that g is not a universal constant but depends on location. The value 9·8 ms^{-2} can be used for the acceleration due to gravity, unless explicitly stated otherwise.
Use calculus in kinematics for motion in a straight line.	To include the use of $v = \frac{\mathrm{d}r}{\mathrm{d}t}$, $a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 r}{\mathrm{d}t^2}$, $r = \int v\,\mathrm{d}t$, $v = \int a\,\mathrm{d}t$, where v, a and r are given in terms of t.

Topics	Guidance
2.2.8 Forces and Newton's laws	
Understand the concept of a force. Understand and use Newton's first law.	
Understand and use Newton's second law for motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2-D vectors).	
Understand and use weight and motion in a straight line under gravity; gravitational acceleration, g, and its value in S.I. units to varying degrees of accuracy. (The inverse square law for gravitation is not required and g may be assumed to be constant, but learners should be aware that g is not a universal constant but depends on location.)	Forces will be constant and will include weight, normal reaction, tension and thrust. To include problems involving lifts. The value $9{\cdot}8\ \text{ms}^{-2}$ can be used for the acceleration due to gravity, unless explicitly stated otherwise.
Understand and use Newton's third law. Equilibrium of forces on a particle and motion in a straight line (restricted to forces in two perpendicular directions or simple cases of forces given as 2-D vectors) Applications to problems involving smooth pulleys and connected particles.	Problems involving particles connected by strings passing over smooth, fixed pulleys or pegs; one particle will be freely hanging and the other particle may be (i) freely hanging, (ii) on a smooth, horizontal plane.
2.2.9 Vectors	
Calculate the magnitude and direction of a vector and convert between component form and magnitude/direction form.	
Use vectors to solve problems in context, including forces.	Does not include kinematics problems.

2.3 A2 UNIT 3

Unit 3: Pure Mathematics B
Written examination : 2 hours 30 minutes
35% of A level qualification
120 marks

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in Unit 1.

The subject content is set out on the following pages. There is no hierarchy implied by the order in which the content is presented, nor should the length of the various sections be taken to imply any view of their relative importance.

Topics	Guidance
2.3.1 Proof	
Proof by contradiction (including proof of the irrationality of $\sqrt{2}$ and the infinity of primes, and application to unfamiliar proofs).	
2.3.2 Algebra and Functions	
Simplify rational expressions, including by factorising and cancelling and by algebraic division (by linear expressions only).	
Sketch curves defined by the modulus of a linear function.	Be able sketch graphs of the form $y = \lvert ax + b \rvert$. To include solving equations and inequalities involving the modulus function.
Understand and use composite functions; inverse functions and their graphs.	Understand and use the definition of a function. Understand and use the domain and range of functions. In the case of a function defined by a formula (with unspecified domain) the domain is taken to be the largest set such that the formula gives a unique image for each element of the set. The notation fg will be used for composition.

Topics	Guidance
Understand the effect of combinations of transformations on the graph of $y = f(x)$, as represented by $y = af(x)$, $y = f(x) + a$, $y = f(x + a)$ and $y = f(ax)$.	
Decompose rational functions into partial fractions (denominators not more complicated than squared linear terms and with no more than 3 terms, numerators constant or linear).	With denominators of the form $(ax + b)(cx + d)$, $(ax+b)(cx+d)(ex+f)$ and $(ax + b)(cx + d)^2$. Learners will **not** be expected to sketch the graphs of rational functions.
Use of functions in modelling, including consideration of limitations and refinements of the models.	
2.3.3 Coordinate geometry in the (x, y) plane	
Understand and use the parametric equations of curves and conversion between Cartesian and parametric forms.	To include finding the equations of tangents and normals to curves defined parametrically or implicitly. Knowledge of the properties of curves other than the circle will **not** be expected.
Use parametric equations in modelling in a variety of contexts.	

Topics	Guidance
2.3.4 Sequences and Series	
Understand and use the binomial expansion of $(a+bx)^n$, for any rational n, including its use for approximation. Be aware that the expansion is valid for $\left\lvert \frac{bx}{a} \right\rvert < 1$ (proof not required).	To include the expansion, in ascending powers of x, of expressions such as $(2-x)^{\frac{1}{2}}$ and $\frac{(4-x)^{\frac{3}{2}}}{(1+2x)}$.
Work with sequences, including those given by a formula for the nth term and those generated by a simple relation of the form $x_{n+1} = f(x_n)$. Increasing sequences, decreasing sequences, periodic sequences.	
Understand and use sigma notation for sums of series.	
Understand and work with arithmetic sequences and series, including the formulae for the nth term and the sum to n terms.	Use of $u_n = a+(n-1)d$. Use and proof of $S_n = \frac{n}{2}[2a+(n-1)d]$ and $S_n = \frac{n}{2}[a+l]$.
Understand and work with geometric sequences and series, including the formulae for the nth term and the sum of a finite geometric series. The sum to infinity of a convergent geometric series, including the use of $\lvert r \rvert < 1$; modulus notation.	Use of $u_n = ar^{n-1}$. Use and proof of $S_n = \frac{a(1-r^n)}{1-r}$. Use of $S_\infty = \frac{a}{1-r}$ for $\lvert r \rvert < 1$.
Use sequences and series in modelling.	

Topics	Guidance
2.3.5 Trigonometry	
Work with radian measure, including use for arc length, area of sector and area of segment.	
Understand and use the standard small angle approximations of sine, cosine and tangent. $\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{\theta^2}{2}$ and $\tan\theta \approx \theta$, where θ is in radians.	
Know and use exact values of $\sin$ and $\cos$ for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi$ and multiples thereof, and exact values of tan for $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \pi$ and multiples thereof.	
Understand and use the definitions of sec, cosec, cot, $\sin^{-1}$, $\cos^{-1}$ and $\tan^{-1}$. Understand the relationships of all of these to sin, cos and tan and understand their graphs, ranges and domains.	
Understand and use $\sec^2\theta \equiv 1 + \tan^2\theta$ and $\operatorname{cosec}^2\theta \equiv 1 + \cot^2\theta$.	The solution of trigonometric equations such as $\sec^2\theta + 5 = 5\tan\theta$.
Understand and use double angle formulae. Use of formulae for $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$. Understand geometric proofs of these formulae.	Use of these formulae to solve equations in a given range, e.g. $\sin 2\theta = \sin\theta$, Applications to integration, e.g. $\int \cos^2 x \, dx$.

Topics	Guidance
Understand and use expressions for $a\cos\theta + b\sin\theta$ in the equivalent forms of $r\cos(\theta \pm \alpha)$ or $r\sin(\theta \pm \alpha)$.	Use of these to solve equations in a given range, e.g. $3\cos\theta + \sin\theta = 2$. Application to finding greatest and least values, e.g. the least value of $\dfrac{1}{3\cos\theta + 4\sin\theta + 10}$.
Construct proofs involving trigonometric functions and identities.	
2.3.6 Differentiation	
Differentiation from first principles for $\sin x$ and $\cos x$.	
Understand and use the second derivative as the rate of change of gradient; connection to convex and concave sections of curves, and points of inflection.	Points of inflection to include stationary and non-stationary points.
Differentiate e^{kx}, a^{kx}, $\sin kx$, $\cos kx$, $\tan kx$, and related sums, differences and constant multiples. Understand and use the derivative of $\ln x$.	
Apply differentiation to find points of inflection.	
Differentiate using the product rule, the quotient rule and the chain rule, including problems involving connected rates of change and inverse functions.	To include the use of $\dfrac{dy}{dx} = \dfrac{1}{\left(\dfrac{dx}{dy}\right)}$.
Differentiate simple functions and relations defined implicitly or parametrically, for first derivative only.	
Construct simple differential equations in pure mathematics.	

2.3.7 Integration	
Integrate e^{kx}, $\frac{1}{x}$, $\sin kx$, $\cos kx$ and related sums, differences and constant multiples.	Use of the results: 1) if $\int f(x)dx = F(x) + k$ then $\int f(ax+b)dx = \frac{1}{a}F(ax+b) + c.$ 2) $\int f'(g(x))g'(x)dx = f(g(x)) + c$
Use a definite integral to find the area between two curves.	
Understand and use integration as the limit of a sum.	
Carry out simple cases of integration by substitution and integration by parts. Understand these methods as the reverse processes of the chain rule and the product rule respectively. Integration by substitution includes finding a suitable substitution and is limited to cases where one substitution will lead to a function which can be integrated. Integration by parts includes more than one application of the method but excludes reduction formulae.	
Integrate using partial fractions that are linear in the denominator.	
Evaluate the analytical solution of simple first order differential equations with separable variables, including finding particular solutions. (Separation of variables may require factorisation involving a common factor.)	Questions will be set in pure mathematics only.

2.3.8 Numerical Methods	
Locate roots of $f(x) = 0$ by considering changes in sign of $f(x)$ in an interval of x in which $f(x)$ is sufficiently well-behaved. Understand how change of sign methods can fail.	
Solve equations approximately using simple iterative methods; be able to draw associated cobweb and staircase diagrams. Solve equations using the Newton-Raphson method and other recurrence relations of the form $x_{n+1} = g(x_n)$. Understand how such methods can fail.	The iterative formula will be given. Consideration of the conditions for convergence will **not** be required.
Understand and use numerical integration of functions, including the use of the trapezium rule and estimating the approximate area under a curve and limits that it must lie between.	Learners will be expected to use the trapezium rule to estimate the area under a curve and to determine whether it gives an overestimate or an underestimate of the area under a curve. Simpson's rule is excluded.
Use numerical methods to solve problems in context.	To solve problems in context which lead to equations that cannot be solved analytically.

2.4 A2 UNIT 4

Unit 4: Applied Mathematics B
Written examination: 1 hour 45 minutes
25% of A level qualification
80 marks

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in Unit 1, Unit 2 and Unit 3.

The paper will comprise two sections:

Section A: Statistics (40 marks)

Section B: Differential Equations and Mechanics (40 marks)

The total assessment time of 1 hour 45 minutes can be split between Section A and Section B as candidates deem appropriate.

The subject content is set out on the following pages. There is no hierarchy implied by the order in which the content is presented, nor should the length of the various sections be taken to imply any view of their relative importance.

Topics	Guidance
STATISTICS	
2.4.1 Probability	
Understand and use conditional probability, including the use of tree diagrams, Venn diagrams and two-way tables.	
Understand and use the conditional probability formula: $P(A \cap B) = P(A)\,P(B \mid A) = P(B)\,P(A \mid B)$.	
Modelling with probability, including critiquing assumptions made and the likely effect of more realistic assumptions.	

Topics	**Guidance**
2.4.2 Statistical distributions	
Understand and use the continuous uniform distribution and Normal distributions as models. Find probabilities using the Normal distribution. Link to histograms, mean, standard deviation, points of inflection and the binomial distribution.	Use of calculator / tables to find probabilities. Linear interpolation in tables will **not** be required.
Select an appropriate probability distribution for a context, with appropriate reasoning, including recognising when the continuous uniform or Normal model may not be appropriate.	The distributions from which the selection can be made are: Discrete: binomial, Poisson, uniform Continuous: Normal, uniform
2.4.3 Statistical hypothesis testing	
Understand and apply statistical hypothesis testing to correlation coefficients as measures of how close data points lie to a straight line and be able to interpret a given correlation coefficient using a given *p*-value or critical value. (The calculation of correlation coefficients is excluded.)	Learners will be expected to state hypotheses in terms of ρ, where ρ represents the population correlation coefficient.
Conduct a statistical hypothesis test for the mean of a Normal distribution with known, given or assumed variance, and interpret the results in context.	Learners should know and be able to use the result that if $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ (The proof is excluded.)

Topics	Guidance
DIFFERENTIAL EQUATIONS AND MECHANICS	
2.4.4 Trigonometry	
Use trigonometric functions to solve problems in context, including problems involving vectors, kinematics and forces.	Contexts may include, for example, wave motion as well as problems in vector form which involve resolving directions and quantities in mechanics.
2.4.5 Differentiation	
Construct simple differential equations in context (contexts may include kinematics, population growth and modelling the relationship between price and demand).	To include contexts involving exponential growth and decay.
2.4.6 Integration	
Evaluate the analytical solution of simple first order differential equations with separable variables, including finding particular solutions.	Questions will be set in context. Separation of variables may require factorisation involving a common factor.
Interpret the solution of a differential equation in the context of solving a problem, including identifying limitations of the solution; includes links to kinematics.	

Topics	Guidance
2.4.7 Quantities and units in mechanics	
Understand and use derived quantities and units for moments.	
2.4.8 Kinematics	
Extend, use and derive the formulae for constant acceleration for motion in a straight line to 2 dimensions using vectors.	
Extend the use of calculus in kinematics for motion in a straight line to 2 dimensions using vectors.	To include the use of $\mathbf{v} = \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}$, $\mathbf{a} = \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2}$, $\mathbf{r} = \int \mathbf{v}\,\mathrm{d}t$, $\mathbf{v} = \int \mathbf{a}\,\mathrm{d}t$, where $\mathbf{v}$, $\mathbf{a}$ and $\mathbf{r}$ are given in terms of t.
Model motion under gravity in a vertical plane using vectors; projectiles.	To include finding the speed and direction of motion of the projectile at any point on its path. The maximum horizontal range of a projectile for a given speed of projection. In examination questions, learners may be expected to derive the general form of the formulae for the range, the time of flight, the greatest height or the equation of path. In questions where derivation of formulae has not been requested, the quoting of these formulae will **not** gain full credit. Questions will **not** involve resistive forces.

Topics	Guidance
2.4.9 Forces and Newton's laws	
Extend Newton's second law to situations where forces need to be resolved (restricted to two dimensions).	
Resolve forces in two dimensions. Understand and use the equilibrium of a particle under coplanar forces.	
Understand and use addition of forces; resultant forces; dynamics for motion in a plane.	
Understand and use the $\mathrm{F} \le \mu \mathrm{R}$ model for friction. The coefficient of friction. The motion of a body on a rough surface. Limiting friction and statics.	Forces will be constant and will include weight, friction, normal reaction, tension and thrust. To include motion on an inclined plane. The motion of particles connected by strings passing over smooth, fixed pulleys or pegs; one particle will be freely hanging and the other particle may be on an inclined plane.
2.4.10 Moments	
Understand and use moments in simple static contexts.	To include parallel forces only.
2.4.11 Vectors	
Understand and use vectors in three dimensions.	To include the use of the unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$.
Use vectors to solve problems in context, including forces and kinematics.	Questions will not involve the scalar product.

3 ASSESSMENT

3.1 Assessment objectives and weightings

Below are the assessment objectives for this specification. Learners must demonstrate their ability to:

AO1
Use and apply standard techniques
Learners should be able to:

- select and correctly carry out routine procedures; and
- accurately recall facts, terminology and definitions

AO2
Reason, interpret and communicate mathematically
Learners should be able to:

- construct rigorous mathematical arguments (including proofs);
- make deductions and inference;
- assess the validity of mathematical arguments;
- explain their reasoning; and
- use mathematical language and notation correctly.

AO3
Solve problems within mathematics and in other contexts
Learners should be able to:

- translate problems in mathematical and non-mathematical contexts into mathematical processes;
- interpret solutions to problems in their original context, and, where appropriate, evaluate their accuracy and limitations;
- translate situations in context into mathematical models;
- use mathematical models; and
- evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them.

Approximate assessment objective weightings are shown below as a percentage of the full A level, with AS weightings in brackets.

	AO1	**AO2**	**AO3**	**Total**
AS Unit 1	14% (35%)	5.5% (13.8%)	5.5% (13.8%)	25% (62.5%)
AS Unit 2	6% (15%)	4.5% (11.3%)	4.5% (11.3%)	15% (37.5%)
Total for AS units only	20%	10%	10%	40%
A2 Unit 3	20%	7.5%	7.5%	35%
A2 Unit 4	10%	7.5%	7.5%	25%
Total for A2 units only	30%	15%	15%	60%
Final Total A Level	50%	25%	25%	100%

Use of technology

The use of technology, in particular mathematical and statistical graphing tools and spreadsheets, permeates the study of GCE AS and A Level Mathematics.

A calculator is required for use in all assessments in this specification.

Calculators used must include the following features:

- an iterative function;
- the ability to compute summary statistics and access probabilities from standard statistical distributions.

Calculators must also meet the regulations set out below.

<table>
<tr><td>Calculators must be:

• of a size suitable for use on the desk;
• either battery or solar powered;
• free of lids, cases and covers which have printed instructions or formulas.</td><td rowspan="2">Calculators must not:

• be designed or adapted to offer any of these facilities: -
    • language translators;
    • symbolic algebra manipulation;
    • symbolic differentiation or integration;
    • communication with other machines or the internet;
• be borrowed from another candidate during an examination for any reason;*
• have retrievable information stored in them - this includes:
    • databanks;
    • dictionaries;
    • mathematical formulas;
    • text.</td></tr>
<tr><td>The candidate is responsible for the following:

• the calculator's power supply;
• the calculator's working condition;
• clearing anything stored in the calculator.</td></tr>
</table>

* An invigilator may give a candidate a replacement calculator.

Formula Booklet

A formula booklet will be required in all examinations. This will exclude any formulae listed in Appendix B. Copies of the formula booklet may be obtained from the WJEC.

Statistical Tables

Candidates may use a book of statistical tables for Unit 2 and Unit 4.

The following book of statistical tables is allowed in the examinations:

- Elementary Statistical Tables (RND/WJEC Publications).

4 TECHNICAL INFORMATION

4.1 Making entries

This is a unitised specification which allows for an element of staged assessment.

Assessment opportunities will be available in the summer assessment period each year, until the end of the life of the specification.

Unit 1 and Unit 2 will be available in 2018 (and each year thereafter) and the AS qualification will be awarded for the first time in summer 2018.

Unit 3 and Unit 4 will be available in 2018 (and each year thereafter) and the A level qualification will be awarded for the first time in summer 2018.

A qualification may be taken more than once. However, if any unit has been attempted twice and a candidate wishes to enter the unit for the third time, then the candidate will have to re-enter all units and the appropriate cash-in(s). This is referred to as a 'fresh start'. When retaking a qualification (fresh start), a candidate may have up to two attempts at each unit. However, no results from units taken prior to the fresh start can be used in aggregating the new grade(s).

If a candidate has been entered for but is absent for a unit, the absence does not count as an attempt. The candidate would, however, qualify as a resit candidate.

The entry codes appear below. (To be confirmed)

	Title	Entry codes	
		English-medium	**Welsh-medium**
AS Unit 1	Pure Mathematics A	2300U1	2300N1
AS Unit 2	Applied Mathematics A	2300U2	2300N2
A2 Unit 3	Pure Mathematics B	1300U3	1300N3
A2 Unit 4	Applied Mathematics B	1300U4	1300N4
AS Qualification cash-in		2300QS	2300CS
A level Qualification cash-in		1300QS	1300CS

The current edition of our *Entry Procedures and Coding Information* gives up-to-date entry procedures.

There is no restriction on entry for this specification with any other WJEC AS or A level specification.

4.2 Grading, awarding and reporting

The overall grades for the GCE AS qualification will be recorded as a grade on a scale A to E. The overall grades for the GCE A level qualification will be recorded as a grade on a scale A* to E. Results not attaining the minimum standard for the award will be reported as U (unclassified). Unit grades will be reported as a lower case letter a to e on results slips but not on certificates.

The Uniform Mark Scale (UMS) is used in unitised specifications as a device for reporting, recording and aggregating candidates' unit assessment outcomes. The UMS is used so that candidates who achieve the same standard will have the same uniform mark, irrespective of when the unit was taken. Individual unit results and the overall subject award will be expressed as a uniform mark on a scale common to all GCE qualifications. An AS GCE has a total of 240 uniform marks and an A level GCE has a total of 600 uniform marks. The maximum uniform mark for any unit depends on that unit's weighting in the specification.

Uniform marks correspond to unit grades as follows:

		Unit grade				
Unit weightings	**Maximum unit uniform mark**	**a**	**b**	**c**	**d**	**e**
Unit 1 (25%)	150 (raw mark max=120)	120	105	90	75	60
Unit 2 (15%)	90 (raw mark max=75)	72	63	54	45	36
Unit 3 (35%)	210 (raw mark max=120)	168	147	126	105	84
Unit 4 (25%)	150 (raw mark max=80)	120	105	90	75	60

The uniform marks obtained for each unit are added up and the subject grade is based on this total.

		Qualification grade				
	Maximum uniform marks	**A**	**B**	**C**	**D**	**E**
GCE AS	240	192	168	144	120	96
GCE A level	600	480	420	360	300	240

At A level, Grade A* will be awarded to candidates who have achieved a Grade A (480 uniform marks) in the overall A level qualification and at least 90% of the total uniform marks for the A2 units (324 uniform marks).

APPENDIX A

Mathematical notation

The tables below set out the notation that must be used in the WJEC GCE AS and A Level Mathematics specification. Learners will be expected to understand this notation without the need for further explanation.

AS learners will be expected to understand notation that relates to AS content, and will not be expected to understand notation that relates only to A Level content.

1	**Set Notation**	
1.1	$\in$	is an element of
1.2	$\notin$	is not an element of
1.3	$\subseteq$	is a subset of
1.4	$\subset$	is a proper subset of
1.5	$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
1.6	$\{x: \ldots\}$	the set of all x such that ...
1.7	$\mathrm{n}(A)$	the number of elements in set A
1.8	$\emptyset$	the empty set
1.9	$\boldsymbol{\varepsilon}$	the universal set
1.10	A'	the complement of the set A
1.11	$\mathbb{N}$	the set of natural numbers, $\{1, 2, 3, \ldots\}$
1.12	$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \ldots\}$
1.13	$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \ldots\}$
1.14	$\mathbb{Z}_0^+$	the set of non-negative integers, $\{0, 1, 2, 3, \ldots\}$
1.15	$\mathbb{R}$	the set of real numbers
1.16	$\mathbb{Q}$	the set of rational numbers $\left\{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{Z}^+\right\}$
1.17	$\cup$	union
1.18	$\cap$	intersection
1.19	(x, y)	the ordered pair x, y
1.20	$[a, b]$	the closed interval $\{x \in \mathbb{R}: a \leq x \leq b\}$
1.21	$[a, b)$	the interval $\{x \in \mathbb{R}: a \leq x < b\}$
1.22	$(a, b]$	the interval $\{x \in \mathbb{R}: a < x \leq b\}$
1.23	(a, b)	the open interval $\{x \in \mathbb{R}: a < x < b\}$
2	**Miscellaneous Symbols**	
2.1	$=$	is equal to
2.2	$\neq$	is not equal to
2.3	$\equiv$	is identical to or congruent to

2.4	$\approx$	is approximately equal to
2.5	∞	infinity
2.6	$\propto$	is proportional to
2.7	$\therefore$	therefore
2.8	$\because$	because
2.9	$<$	is less than
2.10	$\leqslant, \leq$	is less than or equal to, is not greater than
2.11	$>$	is greater than
2.12	$\geqslant, \geq$	is greater than or equal to, is not less than
2.13	$p \Rightarrow q$	p implies q (if p then q)
2.14	$p \Leftarrow q$	p is implied by q (if q then p)
2.15	$p \Leftrightarrow q$	p implies and is implied by q (p is equivalent to q)
2.16	a	first term for an arithmetic or geometric sequence
2.17	l	last term for arithmetic sequence
2.18	d	common difference for an arithmetic sequence
2.19	r	common ratio for a geometric sequence
2.20	S_n	sum to n terms of a sequence
2.21	S_∞	sum to infinity of a sequence
3	**Operations**	
3.1	$a + b$	a plus b
3.2	a - b	a minus b
3.3	$a \times b,\ a\ b,\ a.b$	a multiplied by b
3.4	$a \div b, \frac{a}{b}$	a divided by b
3.5	$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \ldots + a_n$
3.6	$\prod_{i=1}^{n} a_i$	$a_1 \times a_2 \times \ldots \times a_n$
3.7	$\sqrt{a}$	the non-negative square root of a
3.8	$\lvert a \rvert$	the modulus of a
3.9	$n!$	n factorial: $n! = n \times (n-1) \times \ldots \times 2 \times 1$, $n \in \mathbb{N}; 0! = 1$
3.10	$\binom{n}{r},\ ^{n}C_{r},\ _{n}C_{r}$	The binomial coefficient $\frac{n!}{r!(n-r)!}$ for $n, r \in \mathbb{Z}_0^+, r \leqslant n$ or $\frac{n(n-1)\ldots(n-r+1)}{r!}$ for $n \in \mathbb{Q}, r \in \mathbb{Z}_0^+$

4	**Functions**	
4.1	$\mathrm{f}(x)$	the value of the function f at x
4.2	$\mathrm{f} : x \mapsto y$	the function f maps the element x to the element y
4.3	f^{-1}	the inverse function of the function f
4.4	gf	the composite function of f and g which is defined by $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$
4.5	$\lim_{x \to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a
4.6	$\Delta x, \delta x$	an increment of x
4.7	$\dfrac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
4.8	$\dfrac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the n^{th} derivative of y with respect to x
4.9	$\mathrm{f}'(x), \mathrm{f}''(x), \ldots, \mathrm{f}^n\ (x)$	the first, second, …, n^{th} derivatives of $\mathrm{f}\ (x)$ with respect to x
4.10	$\dot{x}, \ddot{x}, \ldots$	the first, second, … derivatives of x with respect to t
4.11	$\int y\ \mathrm{d}x$	the indefinite integral of y with respect to x
4.12	$\int_a^b y\ \mathrm{d}x$	the definite integral of y with respect to x between the limits $x = a$ and $x = b$
5	**Exponential and Logarithmic Functions**	
5.1	e	base of natural logarithms
5.2	e^x, $\exp x$	exponential function of x
5.3	$\log_a x$	logarithm to the base a of x
5.4	$\ln x, \log_{\mathrm{e}} x$	natural logarithm of x
6	**Trigonometric Functions**	
6.1	$\left.\begin{matrix}\sin, \cos, \tan, \\ \mathrm{cosec}, \sec, \cot\end{matrix}\right\}$	the trigonometric functions
6.2	$\left.\begin{matrix}\sin^{-1}, \cos^{-1}, \tan^{-1} \\ \arcsin, \arccos, \arctan\end{matrix}\right\}$	the inverse trigonometric functions
6.3	$°$	degrees
6.4	rad	radians
7	**Vectors**	
7.1	$\mathbf{a}$, $\underline{\mathrm{a}}$, $\utilde{\mathrm{a}}$	the vector $\mathbf{a}$, $\underline{\mathrm{a}}$, $\utilde{\mathrm{a}}$; these alternatives apply throughout section 7
7.2	$\overrightarrow{\mathrm{AB}}$	the vector represented in magnitude and direction by the directed line segment AB
7.3	$\hat{\mathbf{a}}$	a unit vector in the direction of $\mathbf{a}$
7.4	$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
7.5	$\lvert\mathbf{a}\rvert, a$	the magnitude of $\mathbf{a}$

7.6	$\lvert\overrightarrow{\mathrm{AB}}\rvert$, AB	the magnitude of $\overrightarrow{\mathrm{AB}}$
7.7	$\begin{pmatrix} a \\ b \end{pmatrix}$, $a\mathbf{i} + b\mathbf{j}$	column vector and corresponding unit vector notation
7.8	$\mathbf{r}$	position vector
7.9	$\mathbf{s}$	displacement vector
7.10	$\mathbf{v}$	velocity vector
7.11	$\mathbf{a}$	acceleration vector
8	**Probability and Statistics**	
8.1	A, B, C, etc.	events
8.2	$A \cup B$	union of the events A and B
8.3	$A \cap B$	intersection of the events A and B
8.4	$\mathrm{P}(A)$	probability of the event A
8.5	A'	complement of the event A
8.6	$\mathrm{P}(A \mid B)$	probability of the event A conditional on the event B
8.7	X, Y, R, etc.	random variables
8.8	x, y, r, etc.	values of the random variables X, Y, R etc
8.9	$x_1, x_2, \ldots$	values of observations
8.10	$f_1, f_2, \ldots$	frequencies with which the observations $x_1, x_2, \ldots$ occur
8.11	$\mathrm{p}(x)$, $\mathrm{P}(X=x)$	probability function of the discrete random variable X
8.12	$p_1, p_2, \ldots$	probabilities of the values $x_1, x_2, \ldots$ of the discrete random variable X
8.13	$\mathrm{E}(X)$	expectation of the random variable X
8.14	$\mathrm{Var}(X)$	variance of the random variable X
8.15	$\sim$	has the distribution
8.16	$\mathrm{B}(n, p)$	binomial distribution with parameters n and p, where n is the number of trials and p is the probability of success in a trial
8.17	q	$q = 1 - p$ for binomial distribution
8.18	$\mathrm{N}(\mu, \sigma^2)$	Normal distribution with mean μ and variance σ^2
8.19	$\mathrm{Z} \sim \mathrm{N}(0,1)$	standard Normal distribution
8.20	ϕ	probability density function of the standardised Normal variable with distribution $\mathrm{N}(0,1)$
8.21	Φ	corresponding cumulative distribution function
8.22	μ	population mean
8.23	σ^2	population variance
8.24	σ	population standard deviation

8.25	$\overline{x}$	sample mean
8.26	s^2	sample variance
8.27	s	sample standard deviation
8.28	H_0	null hypothesis
8.29	H_1	alternative hypothesis
8.30	r	product moment correlation coefficient for a sample
8.31	ρ	product moment correlation coefficient for a population
8.32	$\mathrm{Po}(\mu)$	Poisson distribution with parameter μ where μ is the mean
8.33	$\mathrm{U}(a, b)$	uniform distribution with parameter a and b, where a and b are the minimum and maximum values, respectively
9	**Mechanics**	
9.1	kg	kilograms
9.2	m	metres
9.3	km	kilometres
9.4	$\mathrm{m/s},\ \mathrm{m\ s^{-1}}$	metres per second (velocity)
9.5	$\mathrm{m/s^2},\ \mathrm{m\ s^{-2}}$	metres per second per second (acceleration)
9.6	F	force or resultant force
9.7	N	newton
9.8	$\mathrm{N\ m}$	newton metre (moment of a force)
9.9	t	time
9.10	s	displacement
9.11	u	initial velocity
9.12	v	velocity or final velocity
9.13	a	acceleration
9.14	g	acceleration due to gravity
9.15	μ	coefficient of friction

APPENDIX B

Mathematical formulae and identities

Learners must be able to use the following formulae and identities for GCE AS and A Level Mathematics, without these formulae and identities being provided, either in these forms or in equivalent forms. These formulae and identities may only be provided where they are the starting point for a proof or as a result to be proved.

Pure Mathematics

Quadratic Equations

$ax^2 + bx + c = 0$ has roots $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Laws of Indices

$a^x a^y \equiv a^{x+y}$
$a^x \div a^y \equiv a^{x-y}$
$(a^x)^y \equiv a^{xy}$

Laws of Logarithms

$x = a^n \Leftrightarrow n = \log_a x$ for $a > 0$ and $x > 0$

$\log_a x + \log_a y \equiv \log_a (xy)$

$\log_a x - \log_a y \equiv \log_a \left(\dfrac{x}{y}\right)$

$k \log_a x \equiv \log_a \left(x^k\right)$

Coordinate Geometry

A straight line graph, gradient m passing through (x_1, y_1) has equation

$$y - y_1 = m(x - x_1)$$

Straight lines with gradients m_1 and m_2 are perpendicular when $m_1 m_2 = -1$

Sequences

General term of arithmetic progression:
$u_n = a + (n-1)d$
General term of a geometric progression:
$u_n = ar^{n-1}$

Trigonometry

In the triangle ABC

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2b\cos A$

Area = $\dfrac{1}{2}ab\sin C$

$\cos^2 A + \sin^2 A \equiv 1$
$\sec^2 A \equiv 1 + \tan^2 A$
$\operatorname{cosec}^2 A \equiv 1 + \cot^2 A$
$\sin 2A \equiv 2\sin A\cos A$
$\cos 2A \equiv \cos^2 A - \sin^2 A$

$\tan 2A \equiv \dfrac{2\tan A}{1 - \tan^2 A}$

Mensuration

Circumference and Area of circle, radius r and diameter d :
$C = 2\pi r = \pi d \quad A = \pi r^2$

Pythagoras' Theorem: In any right-angled triangle where a, b and c are the lengths of the sides and c is the hypotenuse:
$c^2 = a^2 + b^2$

Area of trapezium = $\dfrac{1}{2}(a+b)h,$ where a and b are the lengths of the parallel sides and h is their perpendicular separation.

Volume of a prism = area of cross section × length

For a circle of radius, r , where an angle at the centre of θ radians subtends an arc of length s and encloses an associated sector of area A :
$s = r\theta \quad A = \dfrac{1}{2}r^2\theta$

Calculus and Differential Equations

Differentiation

Function	Derivative
x^n	nx^{n-1}
$\sin kx$	$k\cos kx$
$\cos kx$	$-k\sin kx$
e^{kx}	ke^{kx}
$\ln x$	$\frac{1}{x}$
$f(x)+g(x)$	$f'(x)+g'(x)$
$f(x)g(x)$	$f'(x)g(x)+f(x)g'(x)$
$f(g(x))$	$f'(g(x))g'(x)$

Integration

Function	Integral
x^n	$\frac{1}{n+1}x^{n+1}+c, n \neq -1$
$\cos kx$	$\frac{1}{k}\sin kx + c$
$\sin kx$	$-\frac{1}{k}\cos kx + c$
e^{kx}	$\frac{1}{k}e^{kx}+c$
$\left(\frac{1}{x}\right)$	$\ln\lvert x\rvert + c, x \neq 0$
$f'(x)+g'(x)$	$f(x)+g(x)+c$
$f'(g(x))g'(x)$	$f(g(x))+c$

Area under a curve $= \int_a^b y dx \qquad (y \geq 0)$

Vectors

$|x\mathbf{i} + y\mathbf{i} + z\mathbf{k}| = \sqrt{(x^2 + y^2 + z^2)}$

Mechanics

Forces and Equilibrium

Weight = mass × g
Friction $F \leq \mu R$
Newton's second law in the form: $F = ma$

Kinematics

For motion in a straight line with variable acceleration:

$$v = \frac{dr}{dt} \qquad a = \frac{dv}{dt} = \frac{d^2r}{dt^2}$$

$$r = \int v dt \qquad v = \int a \, \mathrm{d}t$$

Statistics

The mean of a set of data: $\bar{x} = \frac{\sum x}{n} = \frac{\sum fx}{\sum f}$

The standard Normal variable: $Z = \frac{x - \mu}{\sigma}$ where $X \sim N(\mu, \sigma^2)$